FLUID MECHANICS 203
TUTORIAL No.2

APPLICATIONS OF BERNOULLI

On completion of this tutorial you should be able to

- derive Bernoulli's equation for liquids.
- find the pressure losses in piped systems due to fluid friction.
- find the minor frictional losses in piped systems.
- match pumps of known characteristics to a given system.
- derive the basic relationship between pressure, velocity and force..
- solve problems involving flow through orifices.
- solve problems involving flow through Venturi meters.
- understand orifice meters.
- understand nozzle meters.
- understand the principles of jet pumps
- solve problems from past papers.

Let's start by revising basics. The flow of a fluid in a pipe depends upon two fundamental laws, the conservation of mass and energy.

1. PIPE FLOW

The solution of pipe flow problems requires the applications of two principles, the law of conservation of mass (continuity equation) and the law of conservation of energy (Bernoulli's equation)

1.1 CONSERVATION OF MASS

When a fluid flows at a constant rate in a pipe or duct, the mass flow rate must be the same at all points along the length. Consider a liquid being pumped into a tank as shown (fig.1).

The mass flow rate at any section is $m = \rho A u_m$

ρ = density (kg/m^3)
u_m = mean velocity (m/s)
A = Cross Sectional Area (m^2)

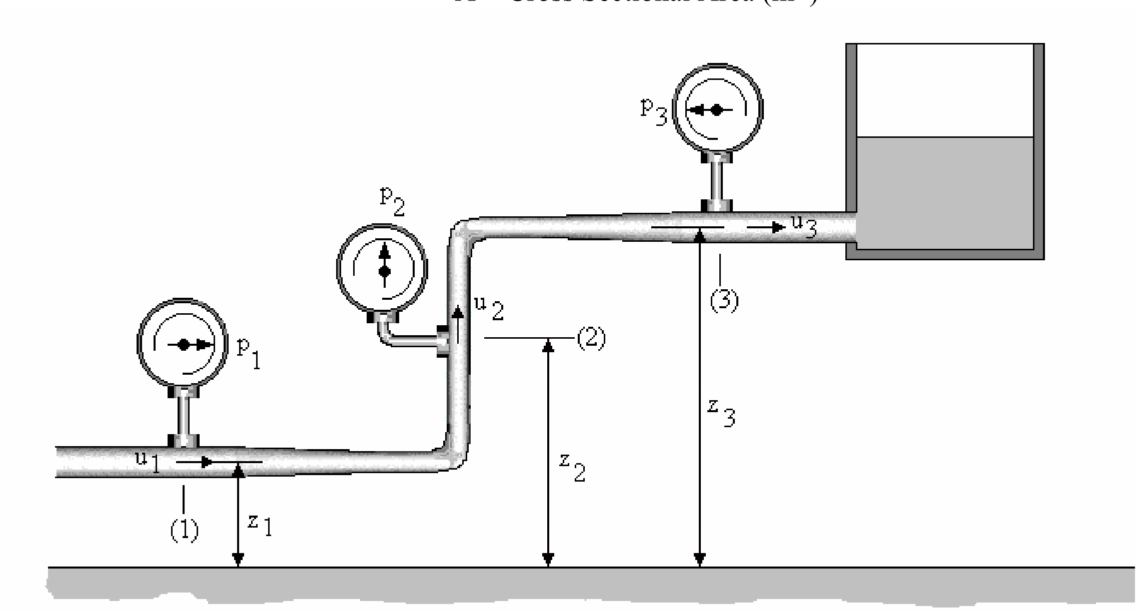

Fig.1.1

For the system shown the mass flow rate at (1), (2) and (3) must be the same so

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2 = \rho_3 A_3 u_3$$

In the case of liquids the density is equal and cancels so

$$A_1 u_1 = A_2 u_2 = A_3 u_3 = Q$$

1.2 CONSERVATION OF ENERGY

ENERGY FORMS

FLOW ENERGY

This is the energy a fluid possesses by virtue of its pressure.

The formula is ***F.E. = pQ Joules***

p is the pressure (Pascals)
Q is volume rate (m^3)

POTENTIAL OR GRAVITATIONAL ENERGY

This is the energy a fluid possesses by virtue of its altitude relative to a datum level.

The formula is ***P.E. = mgz Joules***

m is mass (kg)
z is altitude (m)

KINETIC ENERGY

This is the energy a fluid possesses by virtue of its velocity.

The formula is $K.E. = \frac{1}{2} m u_m^2$ ***Joules***

u_m is mean velocity (m/s)

INTERNAL ENERGY

This is the energy a fluid possesses by virtue of its temperature. It is usually expressed relative to 0^oC. The formula is $U = mc\theta$

c is the specific heat capacity (J/kg ^oC)
θ is the temperature in ^oC

In the following work, internal energy is not considered in the energy balance.

SPECIFIC ENERGY

Specific energy is the energy per kg so the three energy forms as specific energy are as follows.

$F.E./m = pQ/m = p/\rho$ ***Joules/kg***
$P.E/m. = gz$ ***Joules/kg***
$K.E./m = \frac{1}{2} u^2$ ***Joules/kg***

ENERGY HEAD

If the energy terms are divided by the weight mg, the result is energy per Newton. Examining the units closely we have J/N = N m/N = metres.

It is normal to refer to the energy in this form as the energy head. The three energy terms expressed this way are as follows.

$$F.E./mg = p/\rho g = h$$
$$P.E./mg = z$$
$$K.E./mg = u^2/2g$$

The flow energy term is called the pressure head and this follows since earlier it was shown that $p/\rho g = h$. This is the height that the liquid would rise to in a vertical pipe connected to the system.

The potential energy term is the actual altitude relative to a datum.

The term $u^2/2g$ is called the kinetic head and this is the pressure head that would result if the velocity is converted into pressure.

1.3 BERNOULLI'S EQUATION

Bernoulli's equation is based on the conservation of energy. If no energy is added to the system as work or heat then the total energy of the fluid is conserved. Remember that internal (thermal energy) has not been included.

The total energy E_T at (1) and (2) on the diagram (fig.3.1) must be equal so :

$$E_T = p_1Q_1 + mgz_1 + m\frac{u_1^2}{2} = p_2Q_2 + mgz_2 + m\frac{u_2^2}{2}$$

Dividing by mass gives the specific energy form

$$\frac{E_T}{m} = \frac{p_1}{\rho_1} + gz_1 + \frac{u_1^2}{2} = \frac{p_2}{\rho_2} + gz_2 + \frac{u_2^2}{2}$$

Dividing by g gives the energy terms per unit weight

$$\frac{E_T}{mg} = \frac{p_1}{g\rho_1} + z_1 + \frac{u_1^2}{2g} = \frac{p_2}{g\rho_2} + z_2 + \frac{u_2^2}{2g}$$

Since $p/\rho g$ = pressure head h then the total head is given by the following.

$$h_T = h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g}$$

This is the head form of the equation in which each term is an energy head in metres. z is the potential or gravitational head and $u^2/2g$ is the kinetic or velocity head.

For liquids the density is the same at both points so multiplying by ρg gives the pressure form. The total pressure is as follows.

$$p_T = p_1 + \rho gz_1 + \frac{\rho u_1^2}{2} = p_2 + \rho gz_2 + \frac{\rho u_2^2}{2}$$

In real systems there is friction in the pipe and elsewhere. This produces heat that is absorbed by the liquid causing a rise in the internal energy and hence the temperature. In fact the temperature rise will be very small except in extreme cases because it takes a lot of energy to raise the temperature. If the pipe is long, the energy might be lost as heat transfer to the surroundings. Since the equations did not include internal energy, the balance is lost and we need to add an extra term to the right side of the equation to maintain the balance. This term is either the head lost to friction h_L or the pressure loss p_L.

$$h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g} + h_L$$

The pressure form of the equation is as follows.

$$p_1 + \rho gz_1 + \frac{\rho u_1^2}{2} = p_2 + \rho gz_2 + \frac{\rho u_2^2}{2} + p_L$$

The total energy of the fluid (excluding internal energy) is no longer constant.

Note that if a point is a ***free surface*** the pressure is normally atmospheric but if gauge pressures are used, the pressure and pressure head becomes zero. Also, if the surface area is large (say a large tank), the velocity of the surface is small and when squared becomes negligible so the kinetic energy term is neglected (made zero).

WORKED EXAMPLE No. 1

The diagram shows a pump delivering water through as pipe 30 mm bore to a tank. Find the pressure at point (1) when the flow rate is 1.4 dm^3/s. The density of water is 1000 kg/m^3. The loss of pressure due to friction is 50 kPa.

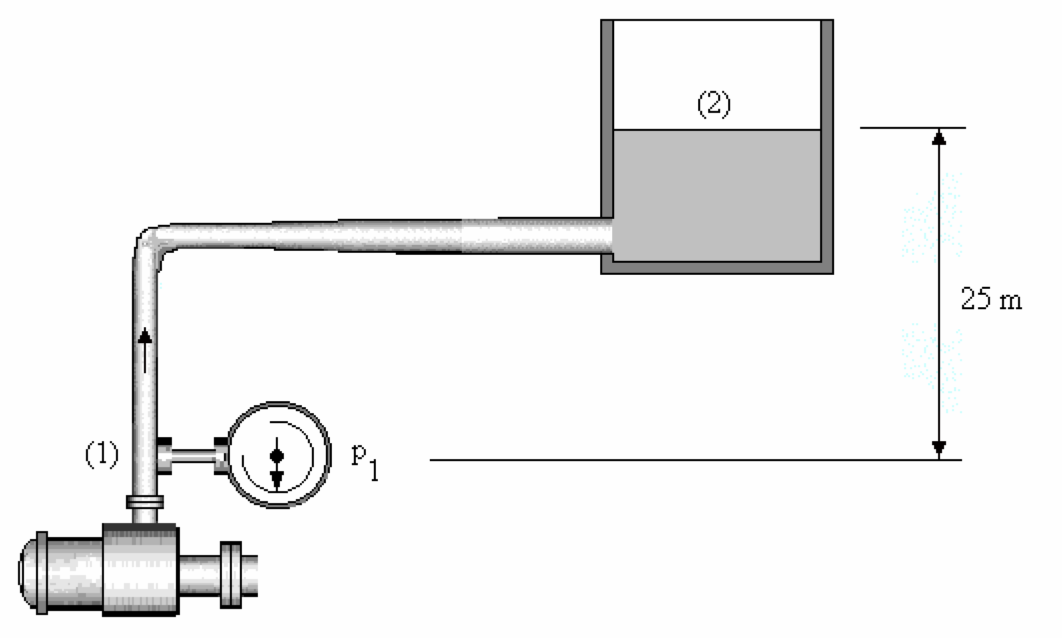

Fig.1.2

SOLUTION

Area of bore A = π x 0.03^2/4 = 706.8 x 10^{-6} m^2.
Flow rate Q = 1.4 dm^3/s = 0.0014 m^3/s
Mean velocity in pipe = Q/A = 1.98 m/s

Apply Bernoulli between point (1) and the surface of the tank.

$$p_1 + \rho g z_1 + \frac{\rho u_1^2}{2} = p_2 + \rho g z_2 + \frac{\rho u_2^2}{2} + p_L$$

Make the low level the datum level and $z_1 = 0$ and $z_2 = 25$.

The pressure on the surface is zero gauge pressure.

P_L = 50 000 Pa

The velocity at (1) is 1.98 m/s and at the surface it is zero.

$$p_1 + 0 + \frac{1000x1.98^2}{2} = 0 + 1000x9.9125 + 0 + 50000$$
$$p_1 = 293.29kPa \text{ gauge pressure}$$

WORKED EXAMPLE 2

The diagram shows a tank that is drained by a horizontal pipe. Calculate the pressure head at point (2) when the valve is partly closed so that the flow rate is reduced to 20 dm^3/s. The pressure loss is equal to 2 m head.

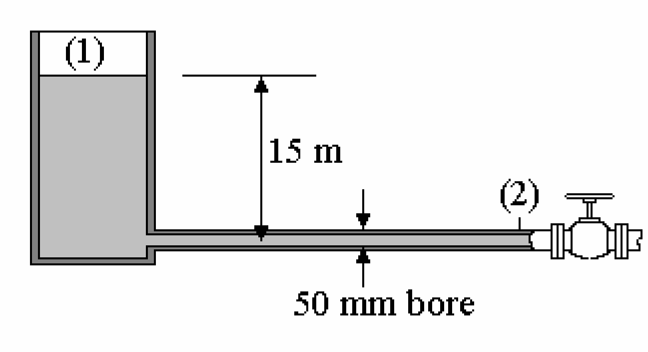

Fig.1.3

SOLUTION

Since point (1) is a free surface, $h_1 = 0$ and u_1 is assumed negligible.

The datum level is point (2) so $z_1 = 15$ and $z_2 = 0$.
Q = 0.02 m3/s
$A_2 = \pi d^2/4 = \pi \times (0.05^2)/4 = 1.963 \times 10^{-3}\ m^2$.
$u_2 = Q/A = 0.02/1.963 \times 10^{-3} = 10.18$ m/s

Bernoulli's equation in head form is as follows.

$$h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g} + h_L$$

$$0 + 15 + 0 = h_2 + 0 + \frac{10.18^2}{2 \text{ x } 9.81} + 2$$

$$h_2 = 7.72\text{m}$$

WORKED EXAMPLE 3

The diagram shows a horizontal nozzle discharging into the atmosphere. The inlet has a bore area of 600 mm^2 and the exit has a bore area of 200 mm^2. Calculate the flow rate when the inlet pressure is 400 Pa. Assume there is no energy loss.

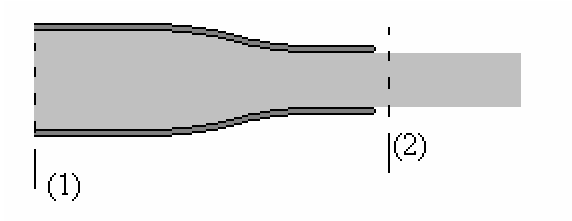

Fig. 1.4

SOLUTION

Apply Bernoulli between (1) and (2)

$$p_1 + \rho g z_1 + \frac{\rho u_1^2}{2} = p_2 + \rho g z_2 + \frac{\rho u_2^2}{2} + p_L$$

Using gauge pressure, p2 = 0 and being horizontal the potential terms cancel. The loss term is zero so the equation simplifies to the following.

$$p_1 + \frac{\rho u_1^2}{2} = \frac{\rho u_2^2}{2}$$

From the continuity equation we have

$$u_1 = \frac{Q}{A_1} = \frac{Q}{600 \times 10^{-6}} = 1666.7Q$$

$$u_2 = \frac{Q}{A_2} = \frac{Q}{200 \times 10^{-6}} = 5\,000\,Q$$

Putting this into Bernoulli's equation we have the following.

$$400 + 1000 \times \frac{(1666.7Q)^2}{2} = 1000 \times \frac{(5000Q)^2}{2}$$

$$400 + 1.389 \times 10^9 Q^2 = 12.5 \times 10^9 Q^2$$

$$400 = 11.11 \times 10^9 Q^2$$

$$Q^2 = \frac{400}{11.11 \times 10^9} = 36 \times 10^{-9}$$

$$Q = 189.7 \times 10^{-6}\ \text{m}^3/\text{s or } 189.7\ \text{cm}^3/\text{s}$$

1.4 HYDRAULIC GRADIENT

Consider a tank draining into another tank at a lower level as shown. There are small vertical tubes at points along the length to indicate the pressure head (h). Relative to a datum, the total energy head is $h_T = h + z + u^2/2g$ and this is shown as line A.

The hydraulic grade line is the line joining the free surfaces in the tubes and represents the sum of h and z only. This is shown as line B and it is always below the line of h_T by the velocity head $u^2/2g$. Note that at exit from the pipe, the velocity head is not recovered but lost as friction as the emerging jet collides with the static liquid. The free surface of the tank does not rise.

The only reason why the hydraulic grade line is not horizontal is because there is a frictional loss h_f. The actual gradient of the line at any point is the rate of change with length $i = \delta h_f/\delta L$

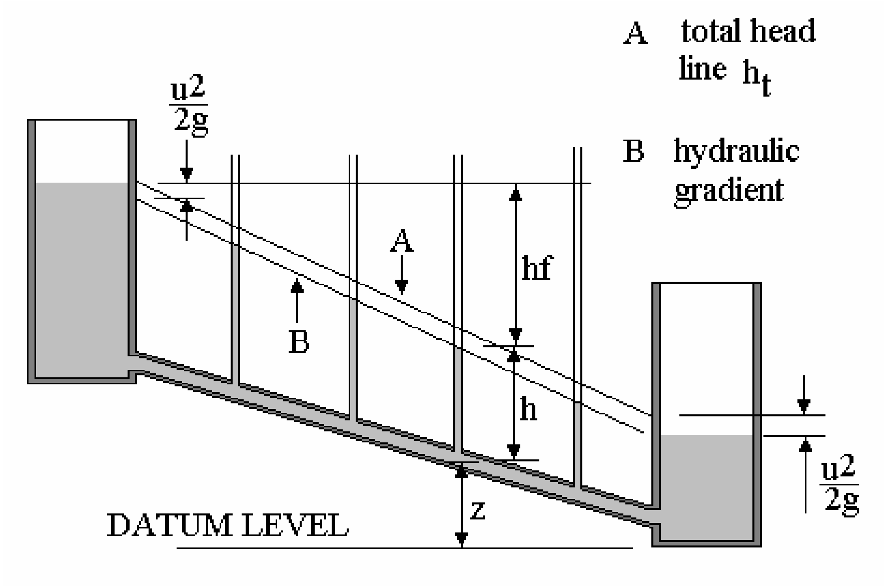

Fig.1.5

SELF ASSESSMENT EXERCISE 1

1. A pipe 100 mm bore diameter carries oil of density 900 kg/m^3 at a rate of 4 kg/s. The pipe reduces to 60 mm bore diameter and rises 120 m in altitude. The pressure at this point is atmospheric (zero gauge). Assuming no frictional losses, determine:

   i. The volume/s (4.44 dm^3/s)
   ii. The velocity at each section (0.566 m/s and 1.57 m/s)
   iii. The pressure at the lower end. (1.06 MPa)

2. A pipe 120 mm bore diameter carries water with a head of 3 m. The pipe descends 12 m in altitude and reduces to 80 mm bore diameter. The pressure head at this point is 13 m. The density is 1000 kg/m^3. Assuming no losses, determine

   i. The velocity in the small pipe (7 m/s)
   ii. The volume flow rate. (35 dm^3/s)

3. A horizontal nozzle reduces from 100 mm bore diameter at inlet to 50 mm at exit. It carries liquid of density 1000 kg/m^3 at a rate of 0.05 m^3/s. The pressure at the wide end is 500 kPa (gauge). Calculate the pressure at the narrow end neglecting friction. (196 kPa)

4. A pipe carries oil of density 800 kg/m^3. At a given point (1) the pipe has a bore area of 0.005 m^2 and the oil flows with a mean velocity of 4 m/s with a gauge pressure of 800 kPa. Point (2) is further along the pipe and there the bore area is 0.002 m^2 and the level is 50 m above point (1). Calculate the pressure at this point (2). Neglect friction. (374 kPa)

5. A horizontal nozzle has an inlet velocity u_1 and an outlet velocity u_2 and discharges into the atmosphere. Show that the velocity at exit is given by the following formulae.

$$u_2 = \{2\Delta p/\rho + u_1^2\}^{½}$$

and

$$u_2 = \{2g\Delta h + u_1^2\}^{½}$$

2 PRESSURE LOSSES IN PIPE SYSTEMS

2 .1 REVIEW OF EARLIER WORK

FRICTION COEFFICIENT

The friction coefficient is a convenient idea that can be used to calculate the pressure drop in a pipe. It is defined as follows.

$$C_f = \frac{\text{Wall Shear Stress}}{\text{Dynamic Pressure}}$$

$$p = \tfrac{1}{2}\rho u_m^2$$

$$\tau_o = \frac{D\Delta p}{4L}$$

$$C_f = \frac{\text{Wall Shear Stress}}{\text{Dynamic Pressure}} = \frac{2D\Delta p}{4L\rho u_m^2}$$

From Poiseuille's equation $\Delta p = \dfrac{32\mu L u_m}{D^2}$ Hence $C_f = \left(\dfrac{2D}{4L\rho u_m^2}\right)\left(\dfrac{32\mu L u}{D^2}\right) = \dfrac{16\mu}{\rho u_m^2 D} = \dfrac{16}{R_e}$

DARCY FORMULA

$$\Delta p = \frac{4C_f L\rho u_m^2}{2D}$$

This is often expressed as a friction head h_f

$$h_f = \frac{\Delta p}{\rho g} = \frac{4C_f L u_m^2}{2gD}$$

This is the Darcy formula. In the case of laminar flow, Darcy's and Poiseuille's equations must give the same result so equating them gives

$$\frac{4C_f L u_m^2}{2gD} = \frac{32\mu L u_m}{\rho g D^2}$$

$$C_f = \frac{16\mu}{\rho u_m D} = \frac{16}{R_e}$$

This is the same result as before for laminar flow.

A formula that gives an approximate answer for any surface roughness is that given by Haaland.

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left\{\frac{6.9}{R_e} + \left(\frac{\varepsilon}{3.71}\right)^{1.11}\right\}$$

This gives a very close model of the Moody chart covered earlier.

WORKED EXAMPLE 4

Determine the friction coefficient for a pipe 100 mm bore with a mean surface roughness of 0.06 mm when a fluid flows through it with a Reynolds number of 20 000.

SOLUTION

The mean surface roughness ε = k/d = 0.06/100 = 0.0006
Locate the line for ε = k/d = 0.0006.
Trace the line until it meets the vertical line at Re = 20 000. Read of the value of C_f horizontally on the left. Answer C_f = 0.0067

Check using the formula from Haaland.

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left\{\frac{6.9}{R_e} + \left(\frac{\varepsilon}{3.71}\right)^{1.11}\right\}$$

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left\{\frac{6.9}{20000} + \left(\frac{0.0006}{3.71}\right)^{1.11}\right\}$$

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left\{\frac{6.9}{20000} + \left(\frac{0.0006}{3.71}\right)^{1.11}\right\}$$

$$\frac{1}{\sqrt{C_f}} = 12.206$$

$$C_f = 0.0067$$

WORKED EXAMPLE 5

Oil flows in a pipe 80 mm bore with a mean velocity of 4 m/s. The mean surface roughness is 0.02 mm and the length is 60 m. The dynamic viscosity is 0.005 N s/m^2 and the density is 900 kg/m^3. Determine the pressure loss.

SOLUTION

$Re = \rho ud/\mu = (900 \times 4 \times 0.08)/0.005 = 57600$

$\varepsilon = k/d = 0.02/80 = 0.00025$

From the chart $C_f = 0.0052$

$h_f = 4C_fLu2/2dg = (4 \times 0.0052 \times 60 \times 4^2)/(2 \times 9.81 \times 0.08) = 12.72$ m

$\Delta p = \rho gh_f = 900 \times 9.81 \times 12.72 = 112.32$ kPa.

2.2 MINOR LOSSES

Minor losses occur in the following circumstances.

i. Exit from a pipe into a tank.
ii. Entry to a pipe from a tank.
iii. Sudden enlargement in a pipe.
iv. Sudden contraction in a pipe.
v. Bends in a pipe.
vi. Any other source of restriction such as pipe fittings and valves.

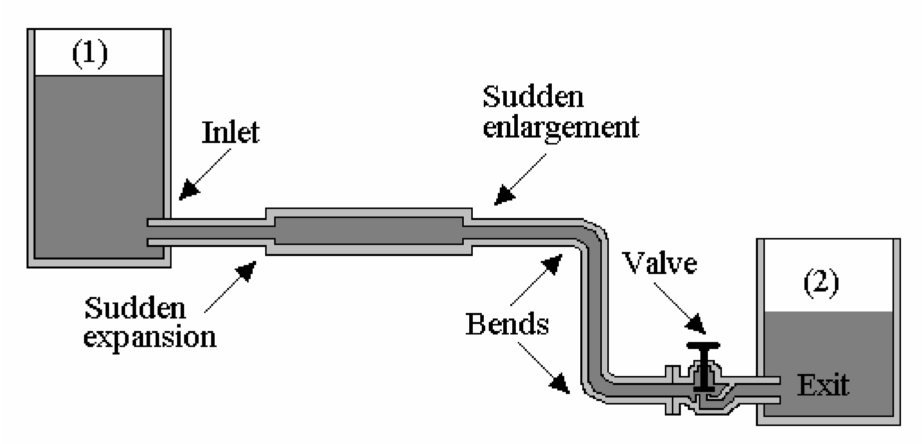

Fig.2.1

In general, minor losses are neglected when the pipe friction is large in comparison but for short pipe systems with bends, fittings and changes in section, the minor losses are the dominant factor.

In general, the minor losses are expressed as a fraction of the kinetic head or dynamic pressure in the smaller pipe.

Minor head loss = k $u^2/2g$ Minor pressure loss = ½ kρu^2

Values of k can be derived for standard cases but for items like elbows and valves in a pipeline, it is determined by experimental methods.

Minor losses can also be expressed in terms of fluid resistance R as follows.

$$h_L = k\frac{u^2}{2} = k\frac{Q^2}{2A^2} = k\frac{8Q^2}{\pi^2 D^4} = RQ^2 \quad \text{Hence } R = \frac{8k}{\pi^2 D^4}$$

$$p_L = k\frac{8\rho g Q^2}{\pi^2 D^4} = RQ^2 \quad \text{hence } R = \frac{8k\rho g}{\pi^2 D^4}$$

Before you go on to look at the derivations, you must first learn about the coefficients of contraction and velocity.

COEFFICIENT OF CONTRACTION Cc

The fluid approaches the entrance from all directions and the radial velocity causes the jet to contract just inside the pipe. The jet then spreads out to fill the pipe. The point where the jet is smallest is called the *VENA CONTRACTA*.

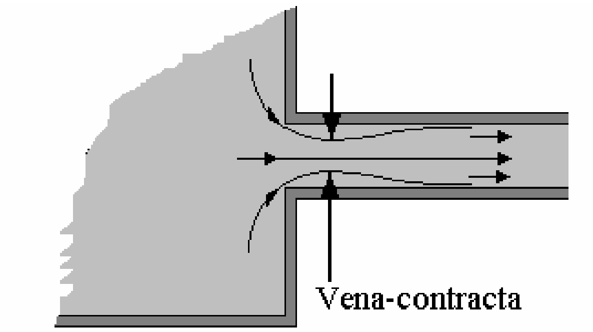

Fig.2.2

The coefficient of contraction C_c is defined as $C_c = A_j/A_o$

A_j is the cross sectional area of the jet and A_o is the c.s.a. of the pipe. For a round pipe this becomes $C_c = d_j^2/d_o^2$.

COEFFICIENT OF VELOCITY C_v

The coefficient of velocity is defined as **C_v = actual velocity/theoretical velocity**

In this instance it refers to the velocity at the vena-contracta but as you will see later on, it applies to other situations also.

EXIT FROM A PIPE INTO A TANK.

The liquid emerges from the pipe and collides with stationary liquid causing it to swirl about before finally coming to rest. All the kinetic energy is dissipated by friction. It follows that all the kinetic head is lost so k = 1.0

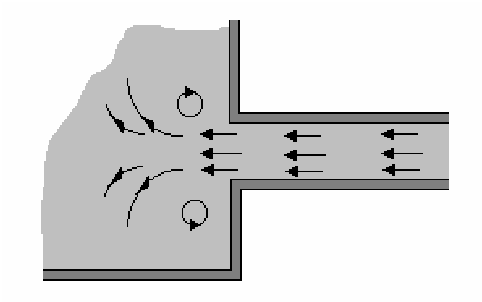

Fig.2.3

ENTRY TO A PIPE FROM A TANK

The value of k varies from 0.78 to 0.04 depending on the shape of the inlet. A good rounded inlet has a low value but the case shown is the worst.

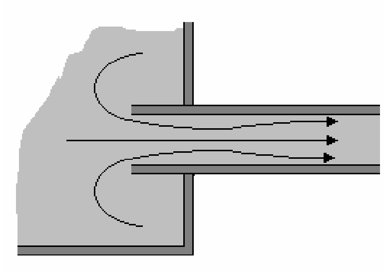

Fig.2.4

SUDDEN ENLARGEMENT

This is similar to a pipe discharging into a tank but this time it does not collide with static fluid but with slower moving fluid in the large pipe. The resulting loss coefficient is given by the following expression.

$$k = \left\{1 - \left(\frac{d_1}{d_2}\right)^2\right\}^2$$

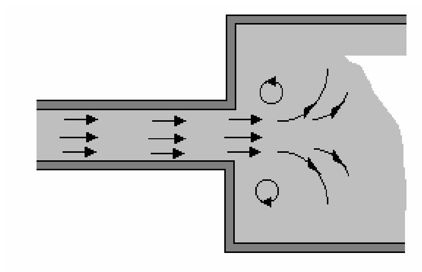

Fig.2.5

SUDDEN CONTRACTION

This is similar to the entry to a pipe from a tank. The best case gives k = 0 and the worse case is for a sharp corner which gives k = 0.5.

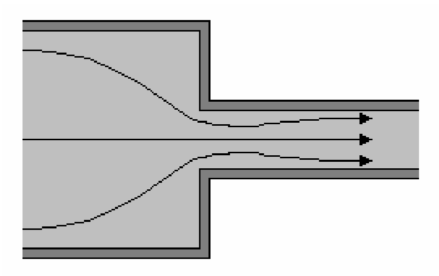

Fig.2.6

BENDS AND FITTINGS

The k value for bends depends upon the radius of the bend and the diameter of the pipe. The k value for bends and the other cases is on various data sheets. For fittings, the manufacturer usually gives the k value. Often instead of a k value, the loss is expressed as an equivalent length of straight pipe that is to be added to L in the Darcy formula.

WORKED EXAMPLE 6

A tank of water empties by gravity through a horizontal pipe into another tank. There is a sudden enlargement in the pipe as shown. At a certain time, the difference in levels is 3 m. Each pipe is 2 m long and has a friction coefficient $C_f = 0.005$. The inlet loss constant is K = 0.3.

Calculate the volume flow rate at this point.

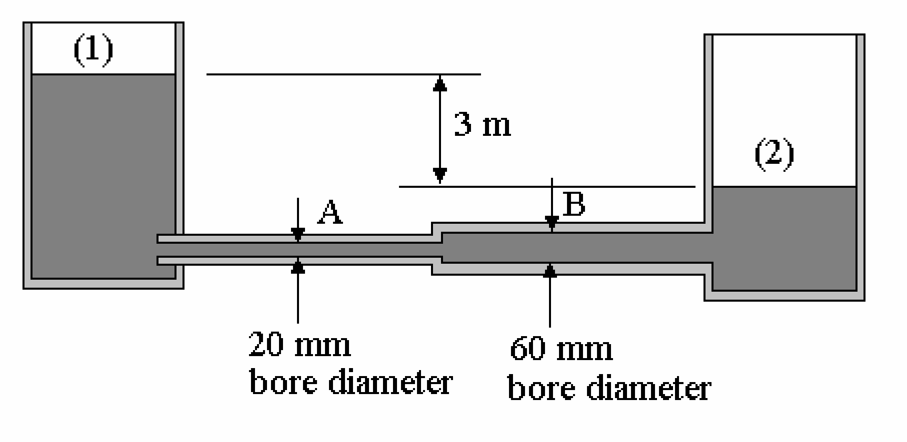

Fig.2.7

SOLUTION

There are five different sources of pressure loss in the system and these may be expressed in terms of the fluid resistance as follows.

The head loss is made up of five different parts. It is usual to express each as a fraction of the kinetic head as follows.

Resistance pipe A $$R_1 = \frac{32C_f L}{gD_A{}^5\pi^2} = \frac{32 \text{ x } 0.005 \text{ x } 2}{g \text{ x } 0.02^5\pi^2} = 1.0328 \text{ x } 10^6 s^2m^{-5}$$

Resistance in pipe B $$R_2 = \frac{32C_f L}{gD_B^5\pi^2} = \frac{32 \text{ x } 0.005 \text{ x } 2}{g \text{ x } 0.06^5\pi^2} = 4.250\text{x}10^3 s^2m^{-5}$$

Loss at entry K=0.3 $$R_3 = \frac{8K}{g\pi^2 D_A{}^4} = \frac{8 \text{ x } 0.3}{g\,\pi^2 \text{x } 0.02^4} = 158\, s^2m^{-5}$$

Loss at sudden enlargement. $$k = \left\{1 - \left(\frac{d_A}{d_B}\right)^2\right\}^2 = \left\{1 - \left(\frac{20}{60}\right)^2\right\}^2 = 0.79$$

$$R_4 = \frac{8K}{g\pi^2 D_A{}^4} = \frac{8\text{x}0.79}{g\pi^2 \text{x } 0.02^4} = 407.7\, s^2m^{-5}$$

Loss at exit K=1 $$R_5 = \frac{8K}{g\pi^2 D_B{}^4} = \frac{8\text{x}1}{g\pi^2 \text{x } 0.06^4} = 63710\, s^2m^{-5}$$

$$h_L = R_1Q^2 + R_2Q^2 + R_3Q^2 + R_4Q^2 + R_5Q^2$$

Total losses. $$h_L = (R_1 + R_2 + R_2 + R_4 + R_5)Q^2$$

$$h_L = 1.101 \text{ x } 10^6 Q^2$$

BERNOULLI'S EQUATION

Apply Bernoulli between the free surfaces (1) and (2)

$$h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g} + h_L$$

On the free surface the velocities are small and about equal and the pressures are both atmospheric so the equation reduces to the following.

$z_1 - z_2 = h_L = 3$

$3 = 1.101 \text{ x } 10^6 Q^2$

$Q^2 = 2.724 \text{ x } 10^{-6}$

$Q = 1.65 \text{ x } 10^{-3} \text{ m}^3\text{/s}$

2.3 SIPHONS

Liquid will siphon from a tank to a lower level even if the pipe connecting them rises above the level of both tanks as shown in the diagram. Calculation will reveal that the pressure at point (2) is lower than atmospheric pressure (a vacuum) and there is a limit to this pressure when the liquid starts to turn into vapour. For water about 8 metres is the practical limit that it can be sucked (8 m water head of vacuum).

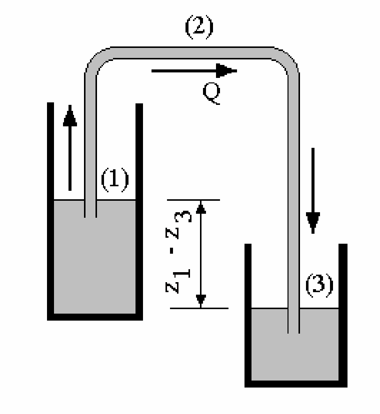

Fig.2.8

WORKED EXAMPLE 7

A tank of water empties by gravity through a siphon. The difference in levels is 3 m and the highest point of the siphon is 2 m above the top surface level and the length of pipe from inlet to the highest point is 2.5 m. The pipe has a bore of 25 mm and length 6 m. The friction coefficient for the pipe is 0.007.The inlet loss coefficient K is 0.7.

Calculate the volume flow rate and the pressure at the highest point in the pipe.

SOLUTION

There are three different sources of pressure loss in the system and these may be expressed in terms of the fluid resistance as follows.

Pipe Resistance $$R_1 = \frac{32C_fL}{gD^5\pi^2} = \frac{32 \text{ x } 0.007 \text{ x } 6}{g \text{ x } 0.025^5\pi^2} = 1.422 \text{ x } 10^6 \text{ s}^2\text{m}^{-5}$$

Entry Loss Resistance $$R_2 = \frac{8K}{g\pi^2D^4} = \frac{8 \text{ x } 0.7}{g\pi^2 \text{x } 0.025^4} = 15.1 \text{ x } 10^3 \text{ s}^2\text{m}^{-5}$$

Exit loss Resistance $$R_3 = \frac{8K}{g\pi^2D^4} = \frac{8\text{x}1}{g\pi^2 \text{x } 0.025^4} = 21.57 \text{ x } 10^3 \text{ s}^2\text{m}^{-5}$$

Total Resistance $R_T = R_1 + R_2 + R_3 = 1.458 \text{ x } 10^6 \text{ s}^2 \text{ m}^{-5}$

Apply Bernoulli between the free surfaces (1) and (3)

$$h_1 + z_1 + \frac{u_1^2}{2g} = h_3 + z_3 + \frac{u_3^2}{2g} + h_L$$
$$0_1 + z_1 + 0 = 0 + z_3 + 0 + h_L$$
$$z_1 - z_3 = h_L = 3$$

Flow rate $$Q = \sqrt{\frac{z_1 - z_3}{R_T}} = \sqrt{\frac{3}{1.458 \text{ x } 10^6}} = 1.434\text{x}10^{-3}\, m^3/s$$

Bore Area $A = \pi D^2/4 = \pi \text{ x } 0.025^2/4 = 490.87 \text{ x } 10^{-6}\ m^2$

Velocity in Pipe $u = Q/A = 1.434 \text{ x } 10^{-3}/490.87 \text{ x } 10^{-6} = 2.922$ m/s

Apply Bernoulli between the free surfaces (1) and (2)

$$h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g} + h_L$$
$$0 + 0 + 0 = h_2 + 2 + \frac{2.922^2}{2g} + h_L$$
$$h_2 = -2 - h_L \frac{2.922^2}{2g} - h_L$$
$$h_2 = -2 + 0.435 - h_L = -2.435 - h_L$$

Calculate the losses between (1) and (2)

Pipe friction Resistance is proportionally smaller by the length ratio.

$R_1 = (2.5/6) \text{ x}1.422 \text{ x } 10^6 = 0.593 \text{ x } 10^6$

Entry Resistance $R_2 = 15.1 \text{ x } 10^3$ as before

Total resistance $R_T = 608.1 \text{ x } 10^3$

Head loss $h_L = R_T\, Q^2 = 1.245$m

The pressure head at point (2) is hence $h_2 = -2.435 - 1.245 = -3.68$ m head

3. MATCHING PUMPS TO A PIPE SYSTEM.

The ideal pump for any given pipe system will produce the required flow rate at the required pressure. The maximum efficiency of the pump will occur at these conditions. These points are considered in detail in a later tutorial.

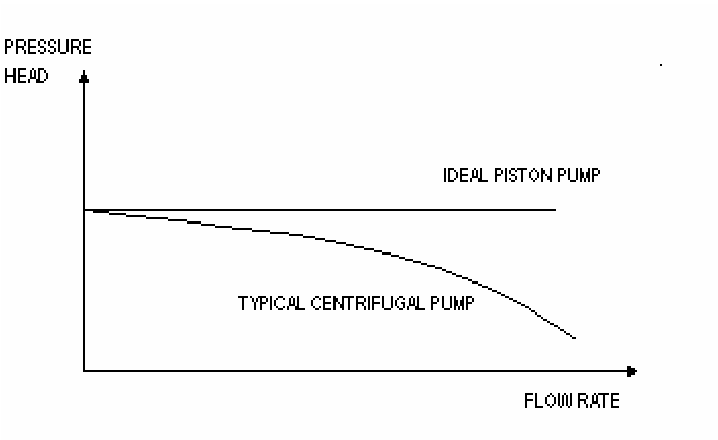

Figure 3.1

The relationship between flow rate Q, pressure head H and efficiency η depend upon the speed but most of all, they depend upon the type of pump. The diagram below shows typical relationships.

The relationship between pressure head and flow rate for a given pipe system is generally one that requires a bigger head for a bigger flow rate. The exact relationship depends upon the losses. If the pump is required to raise the level of the flow, then the required head h is the change in level (lift) plus the losses. The losses are due to pipe friction (and hence the friction factor C_f) , the losses at entry, exit, bends, sudden changes in section and fittings such as valves. The relationship is typically as shown.

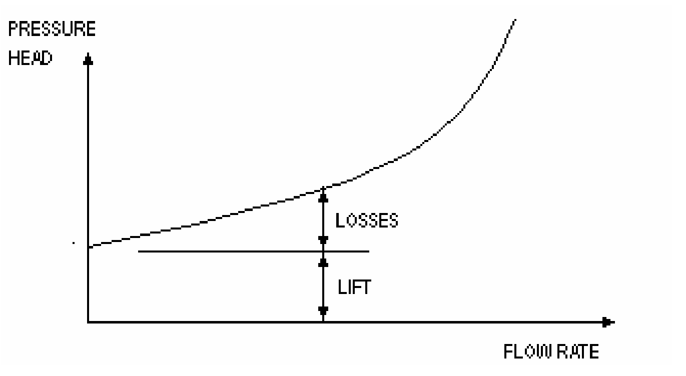

Figure 3.2

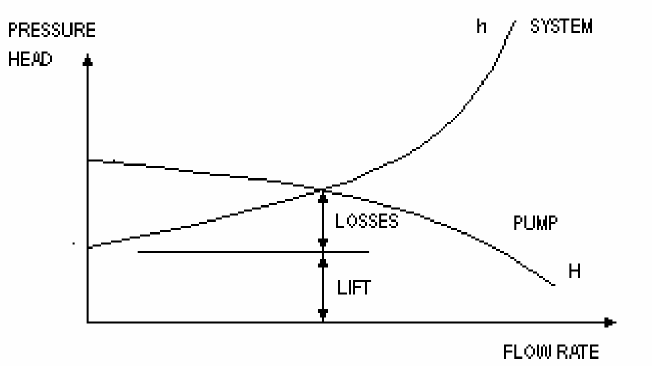

Figure 3.3

If a given pump is to work with a given system, the operating point must be common to each. In other words H = h at the required flow rate.

The solution of problems depends upon finding the relationship between head and flow rate for both the pump and the system and finding the point where the graphs cross.

SELF ASSESSMENT EXERCISE 2

1. A pipe carries oil at a mean velocity of 6 m/s. The pipe is 5 km long and 1.5 m diameter. The surface roughness is 0.8 mm. The density is 890 kg/m^3 and the dynamic viscosity is 0.014 N s/m^2. Determine the friction coefficient from the Moody chart and go on to calculate the friction head h_f.
(Ans. $C_f = 0.0045$ $h_f = 110.1$ m)

2. The diagram shows a tank draining into another lower tank through a pipe. Note the velocity and pressure is both zero on the surface on a large tank. Calculate the flow rate using the data given on the diagram. (Ans. 7.16 dm^3/s)

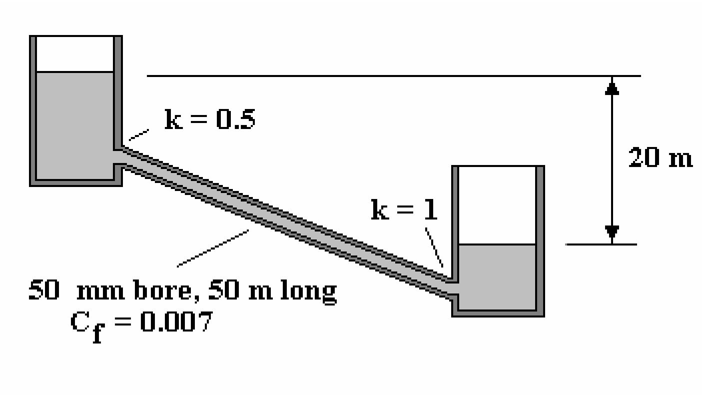

Fig. 3.4

3. Water flows through the sudden pipe expansion shown below at a flow rate of 3 dm^3/s. Upstream of the expansion the pipe diameter is 25 mm and downstream the diameter is 40 mm. There are pressure tappings at section (1), about half a diameter upstream, and at section (2), about 5 diameters downstream. At section (1) the gauge pressure is 0.3 bar.

   Evaluate the following.
   (i) The gauge pressure at section (2) (0.387 bar)
   (ii) The total force exerted by the fluid on the expansion. (-23 N)

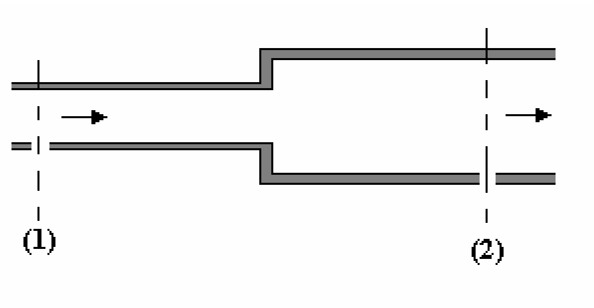

Fig. 3.5

4. A tank of water empties by gravity through a siphon into a lower tank. The difference in levels is 6 m and the highest point of the siphon is 2 m above the top surface level. The length of pipe from the inlet to the highest point is 3 m. The pipe has a bore of 30 mm and length 11 m. The friction coefficient for the pipe is 0.006.The inlet loss coefficient K is 0.6.

   Calculate the volume flow rate and the pressure at the highest point in the pipe.
   (Answers 2.378 dm^3/s and –4.31 m)

5. A domestic water supply consists of a large tank with a loss free-inlet to a 10 mm diameter pipe of length 20 m, that contains 9 right angles bends. The pipe discharges to atmosphere 8.0 m below the free surface level of the water in the tank.

   Evaluate the flow rate of water assuming that there is a loss of 0.75 velocity heads in each bend and that friction in the pipe is given by the Blasius equation $C_f=0.079(Re)^{-0.25}$
   The dynamic viscosity is 0.89×10^{-3} and the density is 997 kg/m^3.
   (0.118 dm^3/s).

6. A pump A whose characteristics are given in table 1, is used to pump water from an open tank through 40 m of 70 mm diameter pipe of friction factor $C_f=0.005$ to another open tank in which the surface level of the water is 5.0 m above that in the supply tank.

   Determine the flow rate when the pump is operated at 1450 rev/min.
   (7.8 dm^3/s)

   It is desired to increase the flow rate and 3 possibilities are under investigation.
   (i) To install a second identical pump in series with pump A.

   (ii) To install a second identical pump in parallel with pump A.

   (iii) To increase the speed of the pump by 10%.

   Predict the flow rate that would occur in each of these situations.

   Head-Flow Characteristics of pump A when operating at 1450 rev/min

Head/m	9.75	8.83	7.73	6.90	5.50	3.83
Flow Rate/(l/s) 4.73	6.22	7.57	8.36	9.55	10.75	

Table 1

7. A steel pipe of 0.075 m inside diameter and length 120 m is connected to a large reservoir. Water is discharged to atmosphere through a gate valve at the free end, which is 6 m below the surface level in the reservoir. There are four right angle bends in the pipe line. Find the rate of discharge when the valve is fully open. (ans. 8.3 dm^3/s).The kinematic viscosity of the water may be taken to be 1.14 x 10^{-6} m^2/s. Use a value of the friction factor C_f taken from table 2 which gives C_f as a function of the Reynolds number Re and allow for other losses as follows.

   at entry to the pipe 0.5 velocity heads.
   at each right angle bend 0.9 velocity heads.
   for a fully open gate valve 0.2 velocity heads.

Re x 10^5	0.987	1.184	1.382
C_f	0.00448	0.00432	0.00419

Table 2

8. (i) Sketch diagrams showing the relationship between Reynolds number, Re, and friction factor, C_f , for the head lost when oil flows through pipes of varying degrees of roughness. Discuss the importance of the information given in the diagrams when specifying the pipework for a particular system.

   (ii) The connection between the supply tank and the suction side of a pump consists of 0.4 m of horizontal pipe , a gate valve one elbow of equivalent pipe length 0.7 m and a vertical pipe down to the tank.

   If the diameter of the pipes is 25 mm and the flow rate is 30 l/min, estimate the maximum distance at which the supply tank may be placed below the pump inlet in order that the pressure there is no less than 0.8 bar absolute. (Ans. 1.78 m)

   The fluid has kinematic viscosity 40×10^{-6} m^2/s and density 870 kg/m^3.

   Assume
   (a) for laminar flow $C_f = 16/(Re)$ and for turbulent flow $C_f = 0.08/(Re)^{0.25}$.

   (b) head loss due to friction is $4C_f V^2L/2gD$ and due to fittings is $KV^2/2g$.

   where K=0.72 for an elbow and K=0.25 for a gate valve.

   What would be a suitable diameter for the delivery pipe ?

4. DIFFERENTIAL PRESSURE DEVICES

Differential pressure devices produce differential pressure as a result of changes in fluid velocity. They have many uses but mainly they are used for flow measurement. In this section you will apply Bernoulli's equation to such devices. You will also briefly examine forces produced by momentum changes.

4.1 GENERAL RELATIONSHIP

Many devices make use of the transition of flow energy into kinetic energy. Consider a flow of liquid which is constrained to flow from one sectional area into a smaller sectional area as shown below.

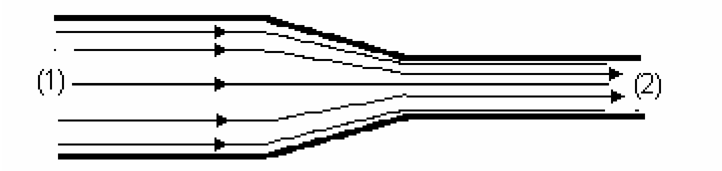

Fig.4.1

The velocity in the smaller bore u_2 is given by the continuity equation as

$$u_2 = u_1A_1/A_2$$

Let $A_1/A_2 = r$ $\quad\quad$ $u_2 = ru_1$

In BS1042 the symbol used is m but r is used here to avoid confusion with mass.

If we apply Bernoulli (head form) between (1) and (2) and ignoring energy losses we have

$$h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g}$$

For a horizontal system $z_1=z_2$ so

$$h_1 + \frac{u_1^2}{2g} = h_2 + \frac{u_2^2}{2g}$$

$$2g(h_1 - h_2) = (u_2^2 - u_1^2) = u_1^2(r^2 - 1)$$

$$u_1 = \sqrt{\frac{2g(h_1 - h_2)}{(r^2 - 1)}}$$

$$Vol/s = Q = A_1u_1 = A_1\sqrt{\frac{2g(h_1 - h_2)}{(r^2 - 1)}}$$

In terms of pressure rather than head we get, by substituting $p= \rho gh$

$$Q = A_1\sqrt{\frac{2\Delta p}{\rho(r^2 - 1)}}$$

To find the mass flow remember $m = \rho Au = \rho Q$

Because we did not allow for energy loss, we introduce a coefficient of discharge C_d to correct the answer resulting in

$$Q = C_d A_1 \sqrt{\frac{2\Delta p}{\rho\left(r^2 - 1\right)}}$$

The value of C_d depends upon many factors and is not constant over a wide range of flows. BS1042 should be used to determine suitable values. It will be shown later that if there is a contraction of the jet, the formula needs further modification.

For a given device, if we regard C_d as constant then the equation may be reduced to :

$$Q = K(\Delta p)^{0.5}$$

where K is the meter constant.

4.2 MOMENTUM and PRESSURE FORCES

Changes in velocities mean changes in momentum and Newton's second law tells us that this is accompanied by a force such that

Force = rate of change of momentum.

Pressure changes in the fluid must also be considered as these also produce a force. Translated into a form that helps us solve the force produced on devices such as those considered here, we use the equation $F = \Delta(pA) + m\,\Delta u$.

When dealing with devices that produce a change in direction, such as pipe bends, this has to be considered more carefully and this is covered in chapter 4. In the case of sudden changes in section, we may apply the formula

$$F = (p_1A_1 + mu_1) - (p_2 A_2 + mu_2)$$

point 1 is upstream and point 2 is downstream.

WORKED EXAMPLE 8

A pipe carrying water experiences a sudden reduction in area as shown. The area at point (1) is 0.002 m^2 and at point (2) it is 0.001 m^2. The pressure at point (2) is 500 kPa and the velocity is 8 m/s. The loss coefficient K is 0.4. The density of water is 1000 kg/m^3. Calculate the following.

i. The mass flow rate.
ii. The pressure at point (1)
iii. The force acting on the section.

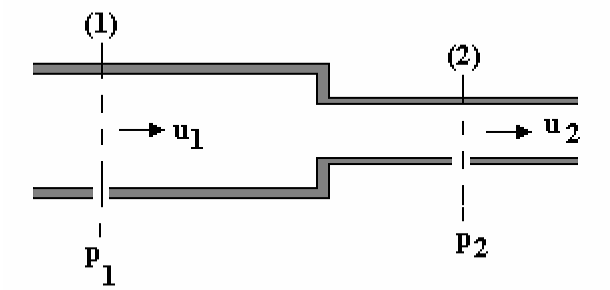

Fig.4.2

SOLUTION

$u_1 = u_2A_2/A_1 = (8 \text{ x } 0.001)/0.002 = 4$ m/s
$m = \rho A_1u_1 = 1000 \text{ x } 0.002 \text{ x } 4 = 8$ kg/s.
$Q = A_1u_1 = 0.002 \text{ x } 4 = 0.008\ m^3/s$
Pressure loss at contraction = $\frac{1}{2}\rho k u_1^2 = \frac{1}{2} \text{ x } 1000 \text{ x } 0.4 \text{ x } 4^2 = 3200$ Pa
Apply Bernoulli between (1) and (2)

$$p_1 + \frac{\rho u_1^2}{2} = p_2 + \frac{\rho u_2^2}{2} + p_L$$

$$p_1 + \frac{1000 \text{ x } 4^2}{2} = 500 \text{ x } 10^3 + \frac{1000 \text{ x } 8^2}{2} + 3200$$

$$p_1 = 527.2 \text{ kPa}$$

$F = (p_1A_1 + mu_1)\text{- }(p_2 A_2 + mu_2)$

$F = [(527.2 \text{ x } 10^3 \text{ x } 0.002) + (8 \text{ x } 4)] – [500 \text{ x } 10^3 \text{ x } 0.001) + (8 \text{ x } 8)]$

F = 1054.4 +32 – 500 – 64

F = 522.4 N

5. SPECIFIC DEVICES

We will now examine specific d.p. devices starting with an orifice. All these devices appear in BS1042

5.1. ORIFICE METERS

When a liquid flows through an orifice it experiences frictional energy loss and a contraction in the diameter of the jet, both of which affect the value of C_d. The diagram below shows this contraction which is due to the fluid approaching the orifice from radial directions and not along the centre line. This makes the velocity of the jet greater than it would otherwise be because of the reduction in area. In addition to this, there is a 2 or 3 % reduction in velocity due to friction. The value of C_d depends upon the sharpness of the orifice edge. In a sharp edged orifice C_d is typically 0.62 but is slightly larger if the sharp edge is replaced by a square edge.

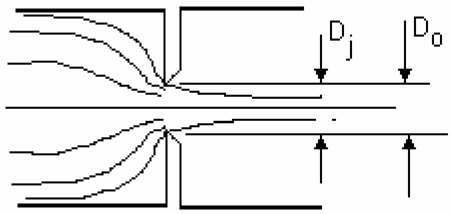

Figure 5.1

5.1.1 COEFFICIENT OF CONTRACTION

The coefficient of contraction is defined as

$$C_c = \text{Area of Jet/Area of Orifice} = A_j/A_o = D_j^2/D_o^2$$

5.1.2 COEFFICIENT OF VELOCITY

The coefficient of velocity is defined as

$$C_v = \text{Actual velocity of jet/theoretical velocity}$$

The theoretical velocity = $(2\Delta p/\rho)^{½}$

It follows that the actual velocity is :

$$u = C_v(2\Delta p/\rho)^{½}$$

5.1.3 COEFFICIENT OF DISCHARGE

The flow rate through the orifice is the product of area and velocity so

$$Q = A_j u = C_c C_v A_o (2\Delta p/\rho)^{½}$$

The product of $C_c C_v$ must be the coefficient of discharge so it follows that

$$C_d = C_c C_v$$

and $$Q = C_d A_o (2\Delta p/\rho)^{½}$$

This formula neglects the approach velocity. The kinetic energy up stream of the orifice is not usually neglected. Let's do the derivation of the flow formula again.

FLOW THROUGH AN ORIFICE

Referring to fig.21, applying Bernoulli between point (1) upstream and the vena-contracta (2) we have

$$p_1 + ½ \rho u_1^2 = p_2 + ½ \rho u_2^2$$
$$p_1 - p_2 = ½ \rho(u_2^2 - u_1^2)$$

$u_1 A_1 = u_2 A_2$ $\qquad u_1 = u_2 A_2/A_1 = u_2 d_2^2/d_1^2$

$A_2/A_0 = C_c = d_2^2/d_0^2$ $\qquad d_2^2 = C_c d_0^2$

$$u_1 = u_2 C_c d_0^2/d_1^2 = u_2 C_c \beta^2$$

$\beta = d_0/d_1$

$$p_1 - p_2 = \Delta p = \frac{1}{2}\rho u_2^2\left(1 - C_c^2\beta^4\right)$$

$$u_2 = \sqrt{\frac{2\Delta p}{\rho\left(1 - C_c^2\beta^4\right)}}$$

This is the velocity at the vena contracta. If friction is taken into account a coefficient of velocity must be used to correct it.

$$u_2 = C_V\sqrt{\frac{2\Delta p}{\rho\left(1 - C_c^2\beta^4\right)}}$$

$Q = A_2 u_2 \quad A_2 = C_c A_0$

$$Q = C_v C_c A_o\sqrt{\frac{2\Delta p}{\rho\left(1 - C_c^2\beta^4\right)}}$$

$$Q = C_d A_o\sqrt{\frac{2\Delta p}{\rho\left(1 - C_c^2\beta^4\right)}}$$

This formula may be rearranged to give the pressure drop if the flow is known.

$$\Delta p = \left(\frac{Q}{C_d A_o}\right)^2\left(1 - C_c^2\beta^4\right)\frac{\rho}{2}$$

The pressure tapping points are normally placed at one pipe diameter upstream and one half pipe diameter downstream in order to get the maximum d.p. However if the maximum value is not important, the d.p. is more easily obtained by the use of corner or flange tappings. The results are still valid but less d.p. is obtained.

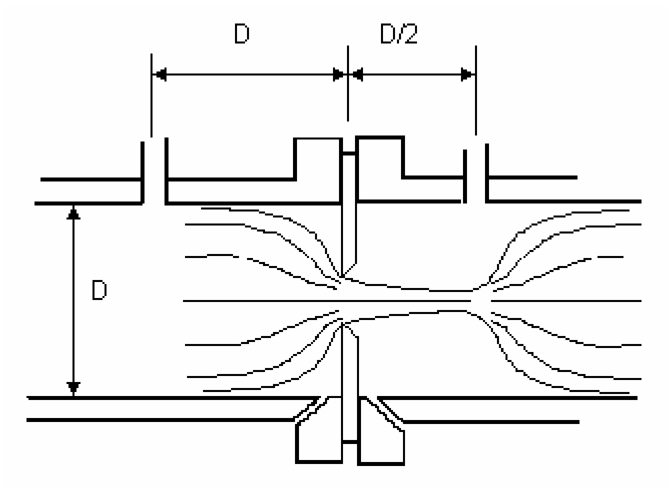

Fig.5.2 showing tapping positions

Figure 5.2 shows how the flow after the orifice must expand to the full bore of the pipe. The velocity in the full bore is less than the jet so the jet must be slowed down. It can only do this by colliding with the slower moving fluid downstream and consequently there is a lot of friction and energy loss in the turbulent mixing taking place. The result is that only a small amount of kinetic energy is reconverted into pressure downstream and the overall pressure loss for the system is high. The loss from the vena contracta (2) to the point downstream where the flow has settled (3) is the loss due to sudden expansion covered earlier and is given by

$$\text{pressure loss due to expansion} = \tfrac{1}{2}\rho(u_2 - u_3)^2$$

Further pressure losses are produced by skin friction and could be estimated. The problem is that the mean velocity is uncertain in the areas near the orifice so it is difficult to apply Darcy's formula.

Figure 5.3 shows the way that pressure changes on approach to and departure from the orifice.

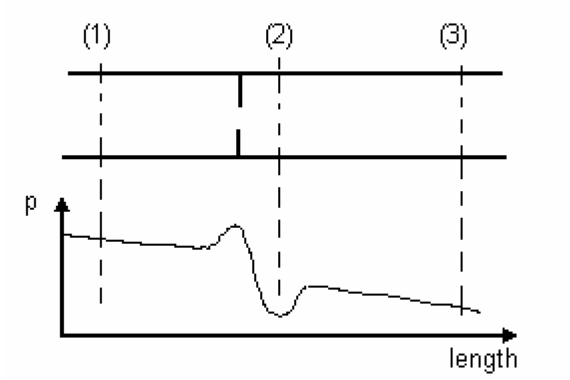

Figure 5.3

WORKED EXAMPLE No.9

The figure shows a sharp edged orifice plate of diameter 20 mm in a horizontal pipe of diameter 25 mm. There are three pressure tappings as follows.

(1) at about 3 pipe diameters upstream of the orifice plate. (2) at half a pipe diameter downstream of the orifice plate and (3) at about 5 pipe diameters downstream of the orifice plate. The tappings read pressures p_1, p_2 and p_3 respectively.

If there is a flow rate of 0.8 x 10^{-3} m^3/s of water at 25^oC, evaluate the pressure differences p_1-p_2 and p_1-p_3. Calculate the % of pressure recovered downstream of the orifice. It may be assumed that the discharge coefficient is 0.64 and the contraction coefficient is 0.74. The density and viscosity for water are usually given on the front of the exam paper. The density is 998 kg/m^3.

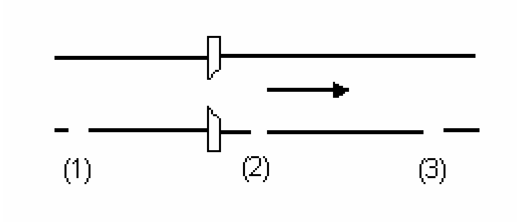

Fig.5.4

SOLUTION

First the pressure drop from 1 to 2. There is friction in the jet so the formula to be used is

$$\Delta p = (Q/C_dA_0)^2(1 - C_c{}^2\beta^4)\rho/2$$

$$A_0 = p \text{ x } 0.02^2/4 = 0.0003142 \text{ m}^2 \quad b = 20/25=0.8$$

$$\Delta p = \{0.0008/(0.64\text{x } 0.0003142)^2\}(1 - 0.74^2\text{x}0.8^4)998/2$$

$$\Delta p = p_1 - p_2 = 6.126 \text{ kPa}$$

This includes the pressure loss due to friction in the jet as well as due to the change in velocity.

$$u_1 = u_3 = 0.0008/(\text{p}\text{x}0.025^2/4\ ) = 1.63 \text{ m/s}$$

$$A_2 = C_c \text{ x p x } 0.02^2/4 = 0.000232 \text{ m}^2$$

$$u_2 = 0.0008/0.000232 = 3.44 \text{ m/s}$$

loss due to sudden expansion = $\rho(u_2 - u_3)^2/2 = 998(3.44 - 1.63)^2/2 = 1.63$ kPa

Now we must find the pressure loss due to friction in the jet.

Ideal jet velocity = u_2/C_v $\qquad$ $C_v = C_d/C_c = 0.64/0.74 = 0.865$

Ideal jet velocity = 3.44/0.865 = 3.98 m/s

Loss of kinetic energy as pressure = $(\rho/2)(3.98^2 - 3.44^2)$ = 1.99 kPa

$p_1 + \rho u_1^2/2 = p_3 + \rho u_3^2/2$

$p_1 - p_3 = (\rho/2)(u_3^2 - u_1^2)$ + losses

$u_3 = u_3$

$p_1 - p_3$ = losses = 1.63 kPa + 1.99 kPa = 3.62 kPa

The pressure regained downstream = 6.126 - 3.62 = 2.5 kPa

The diffuser efficiency = 2.5/6.126 = 41%

5.2. VENTURI METERS

The Venturi Meter is designed to taper down to the throat gradually and then taper out again. No contraction occurs in the flow so $C_c = 1$. The outlet (diffuser) is designed to expand the flow gradually so that the kinetic energy at the throat is reconverted into pressure with little friction. Consequently the coefficient of discharge is much better than for an orifice meter. The overall pressure loss is much better than for an orifice meter.

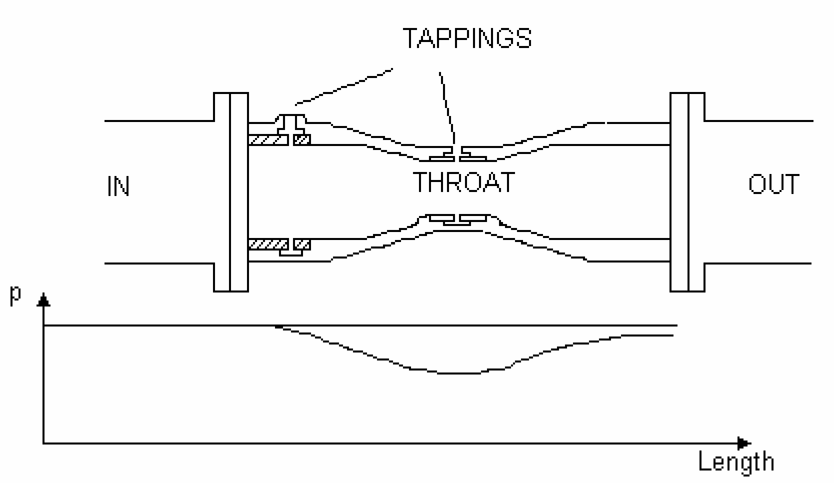

Fig.5.5 showing pressure distribution

If there is no vena-contracta then the flow rate is given by the formula

$$Q = C_d A_1 \sqrt{\frac{2\Delta p}{\rho\left(r^2 - 1\right)}}$$

and $C_d = C_v$ and is about 0.97 for a good meter.

The draw back of the Venturi is the expense involved in the design. The pressure tappings have special inserts in the bore to gather the pressure from around the circumference.

5.3 NOZZLE METER

The nozzle meter is a compromise between the orifice and the venturi. It may be easily fitted in a pipe between flanges with flange or corner tappings. There is no contraction of the jet but there is little pressure recovery downstream. The loss due to sudden expansion occurs down stream. The flow formula is the same as before.

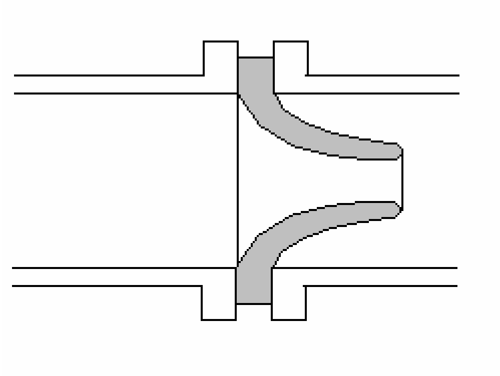

Fig.5.6 Nozzle Meter

WORKED EXAMPLE No.10

A nozzle is 100 mm diameter at inlet and 20 mm diameter at outlet. The coefficient of velocity is 0.97 and there is no contraction of the jet. The jet discharges into the atmosphere. The static pressure at inlet is 300 kPa gauge. The density is 1000 kg/m^3.

Calculate:

a. the velocity at exit.

b. the flow rate.

c. the pressure loss due to friction expressed as a fraction of the dynamic pressure at outlet.

d. the force on the nozzle.

SOLUTION

The velocity at exit when the inlet velocity is not negligible is
$Q = A_1C_d[(2\Delta p/\rho)/(r^2 - 1)]^{0.5}$

$r = A_1/A_2 = d_1^2/d_2^2 = (100/20)^2 = 25$

$C_d = C_v\, C_c = 0.97 \times 1 = 0.97$
$A_1 = (p \times 0.1^2)/4 = 0.00785\ m^2$

hence $Q = 0.97 \times 0.00785\ [(2 \times 300 \times 10^3/1000)/(25^2 - 1)]^{0.5}$
$Q = 0.00747\ m^3/s$

The velocity at inlet = Q/A_1 = 0.00747/0.00785 = 0.951 m/s
The velocity at outlet = Q/A_2 = $0.00747 \times 4/(p \times 0.02^2)$= 23.8 m/s

The dynamic pressure of the jet is $\rho u_2^2/2 = 1000 \times 23.8^2/2$ = 282.7 kPa.

Applying Bernoulli between the inlet (1) and outlet (2) using the pressure form we have
$p_1 - p_2 = \rho u_2^2/2 - \rho u_1^2/2$ + pressure loss to friction
$3 \times 10^5 = (1000/2)(23.8^2 - 0.951^2)$ + pressure loss
$3 \times 10^5 = 2.827 \times 10^5$ + pressure loss

pressure loss = 17.3 kPa

Expressed as a fraction of the dynamic pressure of the jet this is 17.3/ 282.7 or 6.1%.

The force exerted on the water is given by
$F = p_1A_1 + - p_2\, A_2 + mu_1 - mu_2$

We must use gauge pressures to solve this problem because the atmosphere acts on the outer surface of the nozzle. The mass flow is 7.47 kg/s.

$F = 300 \times 10^3 \times 0.00785 - 0 + 7.47(0.951 - 23.8) = 2.18$ kN

The figure is positive which indicates the force is accelerating the water out of the nozzle. The force on the nozzle is the reaction to this and is opposite in direction. Think of a fireman's hose. The force on the nozzle pushes it away from the water like a rocket. The force to accelerate the water must be supplied by those holding it.

6. JET PUMPS

Jet pumps are devices that suck up liquid by the use of a jet discharging into an annular area as shown.

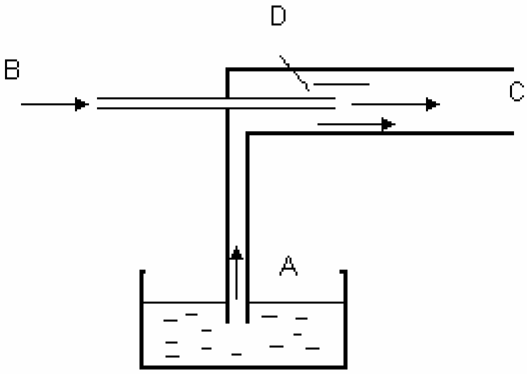

Fig.6.1 A Typical jet Pump.

The solution of jet pump problems requires the use of momentum as well as energy considerations. First apply Bernoulli between A and D and assume no frictional losses. Note that D is a annular area and $u_D = 4Q/\{p(d_1^2-d_2^2)\}$ where d_1 is the diameter of the large pipe and d_2 the diameter of the small pipe.

$$h_A + \frac{u_A^2}{2g} + z_A = h_D + \frac{u_D^2}{2g} + z_D$$

Making A the datum and using gauge pressures we find $h_A = 0$ $u_A = 0$ $z_A = 0$

$$0 = h_D + \frac{u_D^2}{2g} + z_D$$

$$h_D = -\frac{u_D^2}{2g} - z_D$$

From this the head at the point where pipes B and D meet is found.

Next apply the conservation of momentum between the points where B and D join and the exit at C.

$$p_B A_B + \rho Q_B u_B + p_D A_D + \rho Q_D u_D = p_C A_C + \rho Q_C u_C$$

but $p_C = 0$ gauge and $p_B = p_D = p_{(BD)}$ so

$$p_{(BD)} A_{(BD)} + \rho Q_B u_B + \rho Q_D u_D = \rho Q_C u_C$$

where (BD) refers to the area of the large pipe and is the same as A_C.

Next apply conservation of mass $\rho Q_B + \rho Q_D = \rho Q_C$ $Q_B + Q_D = Q_C$

With these equations it is possible to solve the velocity and flow rate in pipe B. The resulting equation is:

$$Q_B^2\left\{\frac{1}{A_B}-\frac{1}{A_C}\right\}-\frac{2Q_BQ_D}{A_C}+\frac{p_BA_C}{\rho}+Q_D^2\left\{\frac{1}{A_D}-\frac{1}{A_C}\right\}=0$$

$aQ_B^2+bQ_B+c=0$ This is a quadratic equation whence

$$Q_B=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

SELF ASSESSMENT EXERCISE 3

Take the density of water to be 997 kg/m^3 throughout unless otherwise stated.

1. A Venturi meter is 50 mm bore diameter at inlet and 10 mm bore diameter at the throat. Oil of density 900 kg/m^3 flows through it and a differential pressure head of 80 mm is produced. Given $C_d = 0.92$, determine the flow rate in kg/s.
(ans. 0.0815 kg/s).

2. A Venturi meter is 60 mm bore diameter at inlet and 20 mm bore diameter at the throat. Water of density 1000 kg/m^3 flows through it and a differential pressure head of 150 mm is produced. Given $C_d = 0.95$, determine the flow rate in dm^3/s.
(ans. 0.515 dm^3/s).

3. Calculate the differential pressure expected from a Venturi meter when the flow rate is 2 dm^3/s of water. The area ratio is 4 and C_d is 0.94. The inlet c.s.a. is 900 mm^2. (ans. 41.916 kPa).

4. Calculate the mass flow rate of water through a Venturi meter when the differential pressure is 980 Pa given $C_d = 0.93$, the area ratio is 5 and the inlet c.s.a. is 1000 mm^2. (ans. 0.266 kg/s).

5. Calculate the flow rate of water through an orifice meter with an area ratio of 4 given C_d is 0.62, the pipe area is 900 mm^2 and the d.p. is 586 Pa. (ans. 0.156 $dm^3/3$).

6. Water flows at a mass flow rate 0f 0.8 kg/s through a pipe of diameter 30 mm fitted with a 15 mm diameter .sharp edged orifice.

   There are pressure tappings (a) 60 mm upstream of the orifice, (b) 15 mm downstream of the orifice and (c) 150 mm downstream of the orifice, recording pressure p_a, p_b and p_c respectively. Assuming a contraction coefficient 0f 0.68, evaluate

   (i) the pressure difference ($p_a - p_b$) and hence the discharge coefficient.
   (21.6 kPa, 0.67)

   (ii)the pressure difference ($p_b - p_c$) and hence the diffuser efficiency.
   (-6.4 kPa, 29.5%)

   (iii) the net force on the orifice plate.
   (10.8 N)

   State any assumption made in your analysis.

7. The figure shows an ejector (or jet pump) which extracts 2 x 10^{-3} m^3/s of water from tank A which is situated 2.0 m below the centre-line of the ejector. The diameter of the outer pipe of the ejector is 40 mm and water is supplied from a reservoir to the thin-walled inner pipe which is of diameter 20 mm. The ejector discharges to atmosphere at section C.

   Evaluate the pressure p at section D, just downstream of the end of pipe B, the velocity in pipe B and the required height of the free water level in the reservoir supplying pipe B. (-21.8 kPa gauge, 12.922 m/s, 6.28 m).

   It may be assumed that both supply pipes are loss free.

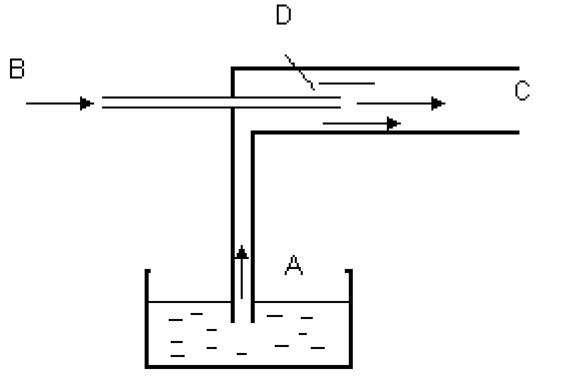

Figure 6.2

8. Discuss the use of orifice plates and venturi-meters for the measurement of flow rates in pipes.

   Water flows with a mean velocity of 0.6 m/s in a 50 mm diameter pipe fitted with a sharp edged orifice of diameter 30 mm. Assuming the contraction coefficient is 0.64, find the pressure difference between tappings at the vena contracta and a few diameters upstream of the orifice, and hence evaluate the discharge coefficient.
   Estimate also the overall pressure loss caused by the orifice plate.
   It may be assumed that there is no loss of energy upstream of the vena contracta.

9. Fig.28 shows an ejector pump BDC designed to lift 2 x 10^{-3} m^3/s of water from an open tank A, 3.0 m below the level of the centre-line of the pump. The pump discharges to atmosphere at C.

   The diameter of thin-walled inner pipe 12 mm and the internal diameter of the outer pipe of the is 25 mm. Assuming that there is no energy loss in pipe AD and there is no shear stress on the wall of pipe DC, calculate the pressure at point D and the required velocity of the water in pipe BD.
   (-43.3 kPa and 20.947 m/s)

   Derive all the equations used and state your assumptions.