below LCL_R are, statistically speaking, out of control; however, they are also desirable because they indicate unusually small variability within the sample which is, after all, one of our main objectives. It is most likely that such small variability is due to special causes.

If the user is convinced that the small variability does indeed represent the operating state of the process during that time, an effort should be made to identify the causes. If such conditions can be created consistently, process variability will be reduced. The process should be set to match those favorable conditions, and the observations should be retained for calculating the revised centerline and the revised control limits for the R-chart.

Step 6: Implement the control charts.

The $\overline{X}$- and R-charts should be implemented for future observations, using the revised centerline and control limits. The charts should be displayed in a conspicuous place where they will be visible to operators, supervisors, and managers. Statistical process control will be effective only if everyone is committed to it—from the operator to the chief executive officer.

Example in our syllabus.

Example 7-1 Consider a process by which coils are manufactured. Samples of size 5 are randomly selected from the process, and the resistance values (in ohms) of the coils are measured. The data values are given in Table 7-2, as are the sample mean $\overline{X}$ and the range R. First, the sum of the ranges is found and then the centerline $\overline{R}$. We have

$$\overline{R} = \frac{\sum_{i=1}^{g} R_i}{g} = \frac{87}{25} = 3.48$$

For a sample of size 5, Appendix A-7 gives $D_4 = 2.114$ and $D_3 = 0$. The trial control limits for the R-chart are calculated as follows:

$$UCL_R = D_4\overline{R} = (2.114)(3.48) = 7.357$$
$$LCL_R = D_3\overline{R} = (0)(3.48) = 0$$

The centerline on the $\overline{X}$-chart is obtained as follows:

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{g} \overline{X}_i}{g} = \frac{521.00}{25} = 20.840$$

Appendix A-7, for $n = 5$, gives $A_2 = 0.577$. Hence, the trial control limits on the $\overline{X}$-charts are

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} = 20.84 + (0.577)(3.48) = 22.848$$
$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R} = 20.84 - (0.577)(3.48) = 18.832$$

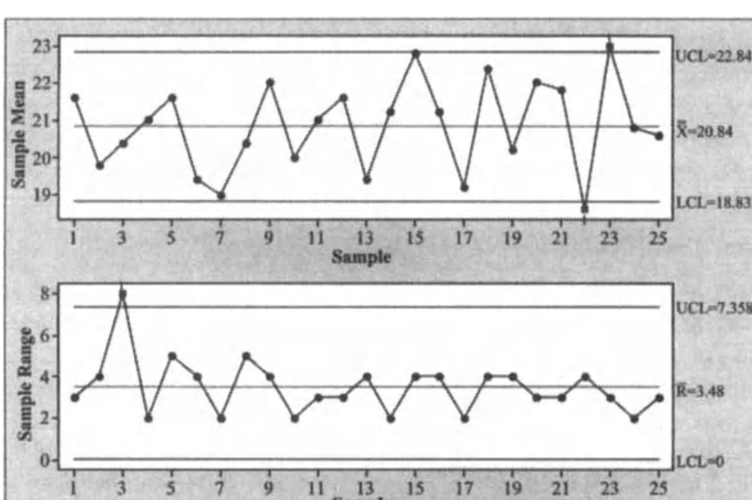
We can use Minitab to construct trial $\overline{X}$- and R-charts for the data in Table 7-2. **Choose Stat > Control Charts > Variables Charts for subgroups > X bar-R.** Indicate whether the subgroups are arranged in a single column or in rows, input, in this case column numbers C1 to C5, since in the worksheet for this example a subgroup is entered as a row across five columns, Click on **X bar-R chart options**, select **Estimate**, and under **Method for estimating standard deviation**, select **Rbar**. Click **OK**. Figure 7-4 shows the Minitab $\overline{X}$- and R-charts with 3σ limits. Observe that sample 3 is above the upper control limit on the R-chart and samples 22 and 23 are below and above the $\overline{X}$ -chart control limit, respectively. When the special causes for these three samples were investigated, operators found that the large value

TABLE 7-2 Coil Resistance Data

Sample	Observation (Ω)	$\bar{X}$	R	Comments
1	20, 22, 21, 23, 22	21.60	3	
2	19, 18, 22, 20, 20	19.80	4	
3	25, 18, 20, 17, 22	20.40	8	New vendor
4	20, 21, 22, 21, 21	21.00	2	
5	19, 24, 23, 22, 20	21.60	5	
6	22, 20, 18, 18, 19	19.40	4	
7	18, 20, 19, 18, 20	19.00	2	
8	20, 18, 23, 20, 21	20.40	5	
9	21, 20, 24, 23, 22	22.00	4	
10	21, 19, 20, 20, 20	20.00	2	
11	20, 20, 23, 22, 20	21.00	3	
12	22, 21, 20, 22, 23	21.60	3	
13	19, 22, 19, 18, 19	19.40	4	
14	20, 21, 22, 21, 22	21.20	2	
15	20, 24, 24, 23, 23	22.80	4	
16	21, 20, 24, 20, 21	21.20	4	
17	20, 18, 18, 20, 20	19.20	2	
18	20, 24, 22, 23, 23	22.40	4	
19	20, 19, 23, 20, 19	20.20	4	
20	22, 21, 21, 24, 22	22.00	3	
21	23, 22, 22, 20, 22	21.80	3	
22	21, 18, 18, 17, 19	18.60	4	High temperature
23	21, 24, 24, 23, 23	23.00	3	Wrong die
24	20, 22, 21, 21, 20	20.80	2	
25	19, 20, 21, 21, 22	20.60	3	
		Sum = 521.00	Sum = 87	

Values may be changed in paper.
Do the same numerical for up to 10 values
with graph as directed by sir.
If this question comes in paper only observations would be given.
you will have to calculate
x bar, x double bar,
R(largest value-Smallest),
R bar (average of all R values)
Draw centre line, upper control limit UCL & lower control limit LCL line.
Draw graph lines on the basis of X Bar values. (10 in our syllabus question)

Use Appendix A-7 given below to find out the value of A2



Only X Chart

NO R Chart needed.

FIGURE 7-4 $\bar{X}$- and R-charts for data on coil resistance using Minitab.

for the range in sample 3 was due to the quality of raw materials and components purchased from a new vendor. Management decided to require the new vendor to provide documentation showing that adequate control measures are being implemented at the vendor's plant and that subsequent deliveries of raw materials and components will conform to standards.

When the special causes for samples 22 and 23 were examined, operators found that the oven temperature was too high for sample 22 and the wrong die was used for sample 23. Remedial actions were taken to rectify these situations.

With samples 3, 22, and 23 deleted, the revised centerline on the R-chart is

$$\bar{R} = \frac{72}{22} = 3.273$$

The revised control limits on the R-chart are

$$\text{UCL}_R = D_4\bar{R} = (2.114)(3.273) = 6.919$$
$$\text{LCL}_R = D_3\bar{R} = (0)(3.273) = 0$$

The revised centerline on the $\bar{X}$-chart is

$$\bar{\bar{X}} = \frac{459}{22} = 20.864$$

The revised control limits on the $\bar{X}$ -chart are

$$\text{UCL}_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} = 20.864 + (0.577)(3.273) = 22.753$$
$$\text{LCL}_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R} = 20.864 - (0.577)(3.273) = 18.975$$

Note that sample 15 falls slightly above the upper control limit on the $\bar{X}$-chart. On further investigation, no special causes could be identified for this sample. So, the revised limits will be used for future observations until a subsequent revision takes place.

Variable Sample Size

So far, our sample size has been assumed to be constant. A change in the sample size has an impact on the control limits for the $\bar{X}$- and R-charts. It can be seen from eqs. (7-7) and (7-9) that an increase in the sample size n reduces the width of the control limits. For an $\bar{X}$-chart, the width of the control limits from the centerline is inversely proportional to the square root of the sample size. Appendix A-7 shows the pattern in which the values of the control chart factors A_2, D_4, and D_3 decrease with an increase in sample size.

Standardized Control Charts

When the sample size varies, the control limits on an $\bar{X}$- and an R-chart will change, as discussed previously. With fluctuating control limits, the rules for identifying out-of-control conditions we discussed in Chapter 6 become difficult to apply—that is, except for Rule 1 (which assumes a process to be out of control when an observation plots outside the control limits). One way to overcome this drawback is to use a standardized control chart. When we standardize a statistic, we subtract its mean from its value and divide this value by its standard deviation. The standardized values then represent the deviation from the mean in units of standard deviation. They are dimensionless and have a mean of zero. The control limits, on a standardized chart are at ± 3 and are therefore constant. It's easier to interpret shifts in the process from a standardized chart than from a chart with fluctuating control limits.

APPENDIX A-7 Factors for Computing Center line and Three-Sigma Control Limits

Observations in Sample, n	$\bar{X}$-Charts			s-Charts						R-Charts						
	Factors for Control Limits			Factors for Center line		Factors for Control Limits				Factors for Center line		Factors for Control Limits				
	A	A_2	A_3	c_4	$1/c_4$	B_3	B_4	B_5	B_6	d_2	$1/d_2$	d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434	1.566
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451	1.548
25	0.600	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459	1.541

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